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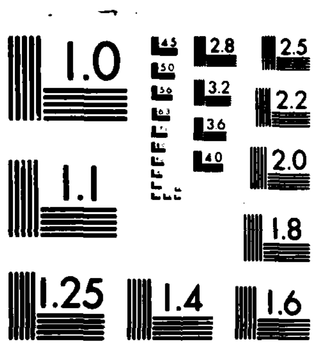

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IFR Results for Repairable Systems

by N.R. Chaganty

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Consider a k-out-of-n system with independent repairable components. Assume that the repair and failure distributions are exponential with parameters $\mu_1,\ldots,\mu_n$ and $\lambda_1,\ldots,\lambda_n$. It is shown that if $\lambda_i-\mu_i = \Delta$ for all i, then the distribution of system life is IFR.

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ABSTRACT

Consider a k-out-of-n system with independent repairable components. Assume that the repair and failure distributions are exponential with parameters $\{\mu_1,\ldots,\mu_n\}$ and $\{\lambda_1,\ldots,\lambda_n\}$ respectively. In this paper ~~we~~ the author shows that if $\lambda_i-\mu_i = \Delta$ for all i then the life distribution of the system is Increasing Failure Rate.

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MATTHEW J. KERPER
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1. Introduction. A k-out-of-n system is one which functions if and only if at least k of the n components function. An n-out-of-n system is known as a series system and an 1-out-of-n system is known as a parallel system. Consider a k-out-of-n system with independent repairable components, each of which has exponentially distributed failure times and repair times. Assume that the exponential failure time distributions have parameters $\lambda_1, \ldots, \lambda_n$ and the exponential repair times have parameters $\mu_1, \ldots, \mu_n$. The study of the life distribution of these systems has received the attention of many authors. Birnbaum-Esary-Marshall [2] have shown that if the components are irrepairable ($\mu_i \equiv 0$, $1 \le i \le n$), then the life distribution of the k-out-of-n system is Increasing Failure Rate on the Average (IFRA). This is true even for general coherent systems. In the special case where λ_i's are all equal to λ and μ_i's are equal to μ, the number of components not working at any time point forms a birth and death process. The results on birth and death processes of Keilson [7], Derman-Ross-Schechner [6], Brown and Chaganty [4] all imply that the life distribution of the k-out-of-n system is Increasing Failure Rate (IFR). More generally without any restrictions on the λ_i's and μ_i's, it was shown by Ross [9], the life distribution of a coherent system is New Better than Used (NBU), from which it follows that the life distribution of the k-out-of-n system is NBU. The theorem of Ross [9] is actually a special case of the result in section 6.6 of Brown and Chaganty [4]. The question whether the

conclusion of Ross [9] can be strengthened to IFRA for a k-out-of-n system remains open. The case k=n=2 failed to yield any counter example. In this paper we show that for a k-out-of-n system if $\lambda_i - \mu_i$ is constant over i, the life distribution of the system is IFR. The motivation and importance for the study of the life distribution of the k-out-of-n system comes from the results of Esary and Proschan [5]. They have shown that the distribution function of the parallel and series systems provide lower and upper bounds respectively for the distribution function of an arbitrary coherent system. The Laplace transform and moments of the parallel system, without any restrictions on λ_i's and μ_i's, and related results were derived in Brown [3].

2. Definitions and Preliminaries. A nonnegative random variable T with distribution F is said to be IFR if $\bar{F}(x+t)/\bar{F}(t)$ is nonincreasing in $-\infty < t < \infty$, for each $x \geq 0$, where $\bar{F} = 1-F$. If the density f exists, this is equivalent to saying that $r(x) = f(x)/\bar{F}(x)$ is nondecreasing in $x \geq 0$. The function r(x) is known as the failure rate. The random variable T is said to be IFRA if $[\bar{F}(x)]^{1/x}$ is nonincreasing in $x \geq 0$ and T is NBU if $\bar{F}(x+y) \leq \bar{F}(x)\,\bar{F}(y)$ for all $x,y \geq 0$. It is well known that

$$\text{IFR} \Rightarrow \text{IFRA} \Rightarrow \text{NBU} ,$$

and none of the reverse implications are true. The properties and usefulness of these three classes of life distributions were well explained in Barlow and Proschan [1].

Consider a coherent system with n components. Let $X_i(t)=0$ if component i is functioning at time t and 1 if it is under repair, i=1,...,n. The vector $X(t)=(X_1(t),\ldots,X_n(t))$ determines the state of the system at time t. To start with we assume that all the components are working, that is, $X(0) = (0,0,\ldots,0)$ with probability 1. Under the asumptions of exponential failure and repair distributions with parameters $0 < \lambda_i < \infty$, $0 < \mu_i < \infty$, $i=1,\ldots,n$, the stochastic process $X = \{X(t),\ t \geq 0\}$ forms a markov process with state space S, consisting of 2^n vectors of 0's and 1's. Let $\{\tau_i,\ i \geq 0\}$ be the successive times of transition for the process X and $Y = \{Y_i,\ i \geq 0\}$ be the successive states visited by X. If $Y_i = \underset{\sim}{x}$, $\underset{\sim}{x} = (x_1,\ldots,x_n) \in S$, then the sojourn interval in $\underset{\sim}{x}$, $[\tau_i,\ \tau_{i+1})$, is exponentially distributed with parameter $\Sigma(x_i\mu_i + (1-x_i)\lambda_i)$ and further Y forms a markov chain with state space S and transition matrix Q, of dimension $2^n \times 2^n$ given by

$$
\begin{aligned}
Q(\underset{\sim}{x},\ \underset{\sim}{y}) &= \frac{\lambda_i}{\Sigma(x_i\mu_i + (1-x_i)\lambda_i)} && \text{if } x_i=0,\ y_i=1,\ x_j=y_j \text{ for } j \neq i \\
&= \frac{\mu_i}{\Sigma(x_i\mu_i + (1-x_i)\lambda_i)} && \text{if } x_i=1,\ y_i=0,\ x_j=y_j \text{ for } j \neq i \\
&= 0 && \text{otherwise.}
\end{aligned}
$$

Let the function $f: S \to N = \{0,1,2,\ldots,n\}$ be defined by $f(\underset{\sim}{x}) = \Sigma x_i$. The function f partitions the state space S into (n+1) parts $A_0,\ldots,A_n$, where $A_\ell = \{\underset{\sim}{x} \in S:\ \Sigma x_i = \ell\}$, $\ell \geq 0$. Let $\delta(0) = \sum_{i=1}^{n} \lambda_i$.

Under the assumption $\lambda_i - \mu_i$ is constant over i, the quantity $\delta(\ell) = \sum_1^{\ell} \mu_i + \sum_{\ell+1}^{n} \lambda_i$ is equal to $\sum_1^{n} (x_i\mu_i + (1-x_i)\lambda_i)$ for all $\underset{\sim}{x} \in A_\ell$. This means that the probability of transition from a state $\underset{\sim}{x} \in A_\ell$ to A_j, $j \neq \ell$, $\ell \geq 0$, is the same for all $\underset{\sim}{x} \in A_\ell$ in the embedded discrete time markov chain Y. Thus by Theorem 6.3.2, page 124, of Kemeny and Snell [8] the lumped process $f(Y) = \{f(Y_i): i \geq 0\}$ is again a markov chain. Hence the process $f(X) = \{f(X(t)): t \geq 0\}$ is a markov process with state space N and successive states visited given by f(Y) at times $\{\tau_i, i \geq 0\}$. Also $[\tau_i, \tau_{i+1})$ is exponentially distributed with parameter $\delta(\ell)$ if $f(Y_i)=\ell$. To compute the transition matrix of f(Y) we proceed as follows. Let $M(\ell)$ be the cardinality of A_ℓ. We can partition the matrix Q as

$$Q = \begin{bmatrix} Q_{00} & Q_{01} & \cdots\cdots & Q_{0n} \\ Q_{10} & Q_{11} & \cdots\cdots & Q_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ Q_{n0} & Q_{n1} & \cdots\cdots & Q_{nn} \end{bmatrix}, \tag{2.2}$$

where $Q_{ij} = (Q(\underset{\sim}{x},\underset{\sim}{y}))$, $\underset{\sim}{x} \in A_i$, $\underset{\sim}{y} \in A_j$, is of order $M(i) \times M(j)$, $i,j \in N$. Let $i_\ell \in N$, $1 \leq \ell \leq m$, $m \geq 1$. Then the joint probabilities of the lumped process f(Y) are given by

$$P[f(Y_\ell) = i_\ell,\ \ell = 1,\ldots, m] = Q_{0i_1} Q_{i_1,i_2} \cdots Q_{i_{m-1},i_m} e_{i_m}, \tag{2.3}$$

where e_{i_m} is a column vector of $M(i_m)$ one's. Note that $Q_{ij}=0$

if $|i-j| \geq 2$. Thus relation (2.3) shows that f(Y) is a birth and death process. The following lemma will be useful to compute the transition probabilities.

<u>Lemma 2.1</u>. Let Q_{ij} be defined as in (2.2). Then

$$Q_{01}Q_{12}\cdots Q_{m-1,m}e_m = \frac{m! \sum\sum_{1\leq i_1<i_2<\cdots<i_m\leq n}\sum \lambda_{i_1}\lambda_{i_2}\cdots\lambda_{i_m}}{\delta(0)\delta(1)\ \ldots\ \delta(m-1)}, \tag{2.4}$$

for $m \geq 1$.

<u>Proof</u>: From (2.3) it follows that the L-H-S of (2.4) is the probability that m components fail in succession. These m components can be choosen in $\binom{n}{m}$ ways. For each selection $(i_1,\ldots,i_m)$ there are m! arrangements of the components and each arrangement has probability $(\lambda_{i_1}\ldots\lambda_{i_m})/\delta(0)\ldots\delta(m-1)$ and hence the R-H-S.

Let $P = (p_{ij})$ be the transition matrix of the markov chain f(Y). Using (2.3) and Lemma 2.1 we can easily verify that for $1 \leq m \leq n-1$,

$$p_{mj} = \frac{(m+1)\sum\sum_{1\leq i_1<\ \ <i_{m+1}\leq n}\sum \lambda_{i_1}\lambda_{i_2}\ \ldots\ \lambda_{i_{m+1}}}{\delta(m)\sum\sum_{1\leq i_1<\cdots<i_m\leq n}\sum \lambda_{i_1}\ \ldots\ \lambda_{i_m}}, \text{ if } j=m+1 \tag{2.5}$$

$$= 1 - p_{m\ m+1} \quad, \text{ if } j=m-1$$

$$= 0 \quad, \text{ otherwise,}$$

and $p_{01} = p_{n\ n-1} = 1$.

The preceding discussion also shows that the markov process $f(X)$ is a birth and death process with birth rates $\{p_{m\ m+1}\delta(m),\ 0\leq m\leq n-1\}$ and death rates $\{p_{m\ m-1}\delta(m),\ 1\leq m\leq n\}$. If the coherent system under consideration is a k-out-of-n system then the life length of the system is just the first passage time to the set $\{k+1, \ldots, n\}$ for the process $f(X)$. Thus the life distribution of the k-out-of-n system is IFR follows from the result in section 6.3 of Brown and Chaganty [4]. Summarizing the above discussion we have the following main result of this paper.

Theorem 2.2. Consider a k-out-of-n system with independent repairable components. Assume that the life distributions are exponential with parameters $\lambda_1,\ldots,\lambda_n$ and repair distributions also exponential with parameters $\mu_1,\ldots,\mu_n$. If $\lambda_i-\mu_i = \Delta$ for all i, then the distribution of the system life is IFR.

Remark 2.3. If $\lambda_1=\lambda_2= \ldots =\lambda_n$ and $\mu_1=\mu_2= \ldots \mu_n$ the above Theorem 2.2 shows that the life distribution of the k-out-of-n system is IFR. This result was mentioned earlier in the introduction.

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